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A TEACHING EXPERIENCE LEARNING OF ALGEBRA EXPRESSIONS FOR FUTURE MATHEMATICS TEACHERS

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ABSTRACT
The experience responds to the didactic sequence about activities developed during the Course Mathematics teaching Methodology at the Universidad Austral (Puerto Montt-Chile). The activities intended to recognize in different concrete materials the applicability in the classes of primary education, in which the students staged. Working with concrete material allowed them to develop visualization skills (visual identification, visual discrimination, position recognition and spatial relations) and visualization processes (Visual Information Interpretation and Visual Processing) in specific tasks. From the initial task, the group showed some difficulties to represent algebraic expressions using geometric representations about the concept of rectangular areas and perimeters. In addition to the learning they were able to develop in using their visual skills and processes, students also learned to communicate better with other peers in the group, to make decisions about problems, and to try to negotiate meanings in and out of mathematics.

INTRODUCTION
The curricular proposal of Universidad Austral de Chile (UACH), for Pedagogy in Mathematics Program is oriented to training the prospective teachers into a balance between the appropriation of disciplinary knowledge and reflections on the teaching and learning processes present and discussed in the field of Mathematics Education. From the Pedagogy in Mathematics Education Program, the future mathematics teacher conceives himself as a professional prepared to practice teaching in Mathematics, with a high scientific-pedagogical level and an adequate general cultural preparation.

One of the courses proposed for the development of pedagogy in Mathematics corresponds to the Mathematics Teaching Methodology, which intends to present different ways of developing mathematical concepts based on the National Mathematical Curriculum framework, as well as in the national framework of Teacher Education in Chile. Among the topics developed in the course is the teaching of Algebra. This communication intends to present, in a brief way, the activities developed during the experiments of teaching to learn algebraic concepts, during the development of the course Teaching Methodology of Mathematics, along with some reflections, after the application of the didactic sequence, under theoretical reference of the Skills and Processes of Visualization.

Images, Skills and Processes in Mathematical Visualization
Before defining the skills involved in the activities, we need to say that we understand visualization as "the set of skills and processes necessary to represent, transform, generalize, communicate and document visual information or reflect upon it" (Gal & Linchevsky, 2010). The present study identifies the skills proposed by Del Grande (1990) and Gutiérrez (1996, 1998); the visualization processes described by Bishop (1983). We can briefly describe visualization skills according to Del Grande (1990) as follows:

- **Visual identification** is the ability to recognize a figure by isolating it from its context.
- **Preservation of perception** and the ability to recognize invariant properties as shape and size, despite the variability given by movement.
- **Recognition of spatial positions** is the ability to relate the position of an object to an observer.
- **Recognition of spatial relationships** is the ability to correctly identify the characteristics of relationships between various objects located in space.
- **Visual discrimination** is the ability to compare two of the most objects identifying their similarities and

differences.

On the other hand, Bishop (1983) refers to the processes of *Interpretation of Figural Information* and *Visual Processing*. The Interpretation of Figural Information (IFI) is the process of interpreting visual representations to extract information from them, and Visual Processing (PV) is the inverse process of IFI. According to Presmeg (1986), we can define different types of images, but the general definition of images corresponds to the mental scenes that describe visual or spatial formation of an object without requiring its presence or other external representation. The images are classified in pictorial, dynamic, kinetic and formulas.

Context of experience

The experience was developed with the group of 15 students of the sixth semester Mathematics Teaching Methodology, as part of the Mathematics Pedagogy Program at Universidad Austral de Chile (Puerto Montt). For the development of the Course, the presentation, use and construction of didactic-methodological resources for the teaching of mathematics was sought. That is to say, some classes were developed under readings of the traditional theoretical references of the didactics of mathematics, and others, they were developed by means of activities and elaboration of lesson plans for the concepts or mathematical contents that they found to be 'difficult' for the Teaching at school.

In this sense, the proposal of teaching algebraic expressions, responds to the needs identified by the students, during their approach to practices in schools. As a consequence, the activities proposed in the course were oriented to the re-signification of the contents of the classroom. One of the objectives of the planned activities is to offer situations that allow the students to reflect, as future teachers, situations that they can plan in the classroom and that permit to interrelate contents, to problematize the practices of teaching to learn as well as to reflect on what happened.

Didactic proposal for Methodology of Mathematics Teaching

For the work with algebraic expressions the material called the Box of Polynomials was used. For which there are several proposals made in different countries (ACEVEDO, 2004, MENDEZ, 2012, SOTO, et al., 2009). This material, which was created with tangible elements in Colombia and later developed as software with its same characteristics. The material intends to develop the concepts as similar terms, operations with polynomials and notable products of algebraic expressions, under the concept of rectangular surface areas that operate with each other, on a tray divided in red and blue quadrants, as shown in figure 1.

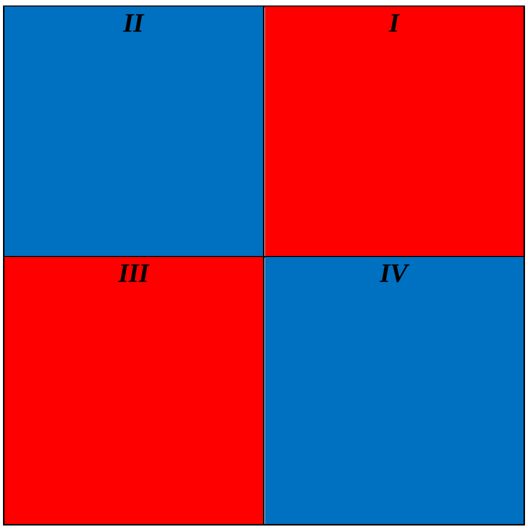

Figure 1. Red-blue plane
Source: Personal collection

The didactic sequence of activities described to proceed, it presents a work proposal using geometric representations for algebraic expressions, in which the students involve the abilities and visualization processes, as well as the images (pictorial, kinetics, dynamics and of formulas).

A) *Construction of the material.* In order to construct the material, I asked the students to make the board at home, with indications of measurements and colors as shown in figure 1. Later, I asked the students to bring to the class, at least 10 sheets of different colors, a ruler, And pens. In figure 2 some rectangles are presented on the board constructed by a group of students of the class of the Course.

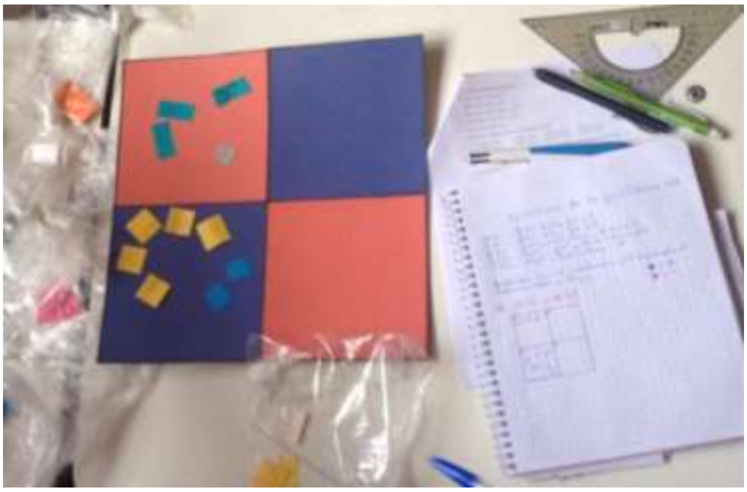

Figure 2. Concrete representations and formulas
Source: Personal collection

The activity consisted in constructing as many rectangles as possible with the measurements of the sides (1, x, x2 and y) that were given on a separate sheet. It should be noted that 1 did not correspond to the measure of 1 cm in the ruler, and that the measurement of the x, x2 and y sides were not multiples of each other.

B) *Find the value of the area for each rectangle.* In this second activity, the students found 15 different rectangles that represented with different colors. Table 1 shows the relationship between the areas and the values of the sides of the rectangles.

Table 1. Relation of surfaces

Area	1	x	x^2	x^2	x^3	x^3	x^4	x^4	x^5	x^6	xy	y	y^2	x^2y	x^3y
Side 1	1	1	x	1	x^2	x^3	x^2	x^3	x^3	x^2	x	y	y	x^2	x^3
Side 2	1	x	x	x^2	x	1	x^2	x	x^2	x^2	y	1	y	y	y

Source: Authors' Construction

C) *Handling of materials.* After constructing the rectangles and identifying their areas, the students manipulated the material, trying to find similar terms, that is, the similarity was worked under the concept of equivalent area, as shown in figure 3. For example, students found that there were two rectangles of different shape, but with the area equal to x2. They also found similarity to x4. This concept is important for recognizing similar terms and doing operations.

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Figure 3. Interactions and records
Source: Personal collection

D) *Representation of polynomials.* After the recognition of similar figures (rectangles of the same area) the rules of use of the material to add and subtract were presented. That is, to perform an operation (addition or subtraction); we must represent the polynomials using the index cards, so that the indexes of the first polynomial remain in quadrant II and III, and in the quadrants I and IV, those of the second polynomial. Note that in Figure 4, where the polynomial M (x) = 5x4-3x + 2x + 1, is represented in quadrant II (positive terms on blue) and III (negative terms on red). In the same way, the polynomial N (x) = 8x3-4x2-3x + 5 was represented using the quadrants I (red) and IV (blue).

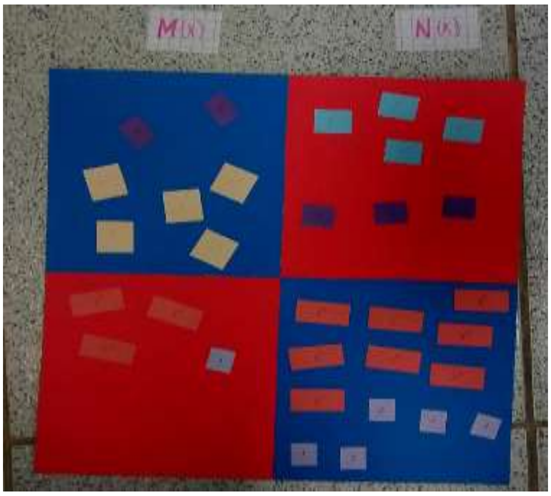

Figure 4. Representation of the polynomials
Source: Personal collection

E) *Addition of polynomials.* In order to develop the indicated sum of the polynomials M (x) and N (x), all the terms that are in the quadrants of the same color are grouped. That is, translate the terms of quadrants II and IV, remaining only in quadrant II (or quadrant IV), and those of red color in quadrant I (or quadrant III), as shown in figure 5.

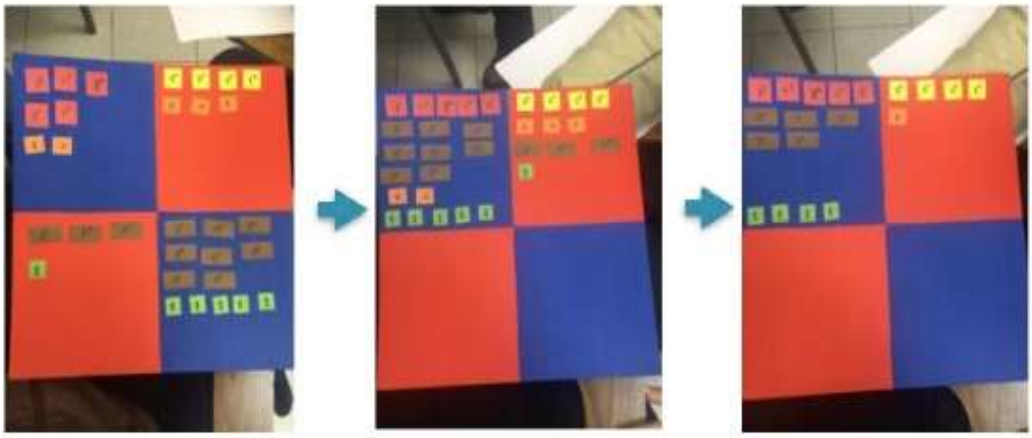

Figure 5. Sum of polynomials
Source: Personal collection

F) *Subtraction of polynomials.* In subtraction of polynomials the rule varies, since the subtraction is worked as a sum. So you need to change the sign of the second polynomial, for which the color change is used. That is, the subtraction between the polynomials M (x) - N (x), being equivalent to the following representation: M (x) + [- N (x)]. Thus, the students change the second polynomial, to which they must translate the terms from quadrant I to II and from quadrant IV to III as shown in figure 6.

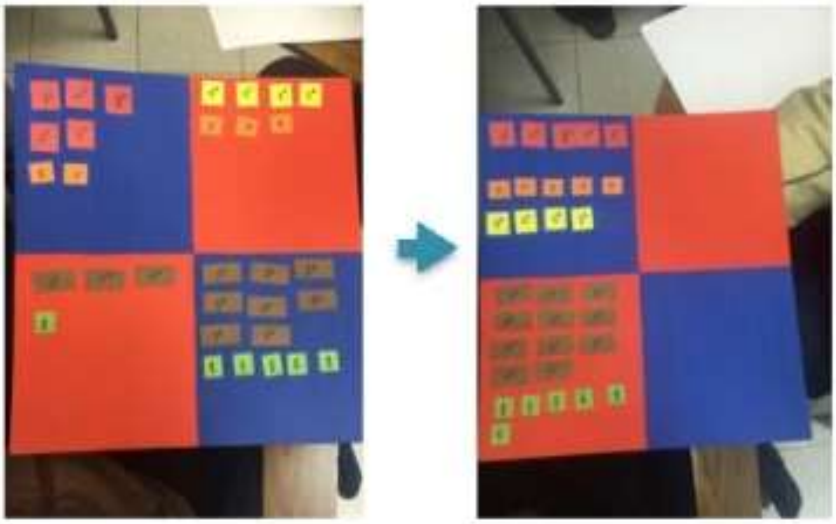

Figure 6. Subtraction of polynomials
Source: Personal collection

G) *Different forms of representation.* Initially it is proposed for the students to explore the operations with the use of the concrete material in a graphical way (kinetic pictorial images) or as algebraic expressions (formulas images).

Figure 4. Representation of formulas
Source: Personal collection

H) *Dynamic records.* At the initiative of the students, some records were made using the cell phone or the tablets themselves.

Figure 5. Interactions and records
Source: Personal collection

It is necessary to use representations on white sheets (as much as possible), which allow the development of visualization skills such as visual discrimination, recognition of spatial positions and spatial relationships, as shown in figure 6. Also, the Use of this type of medium, will allow when teaching does not draw geometric figures prototypically on the plane of the sheet or the blackboard.

Figure 6. Records on white sheets
Source: Personal collection

RESULTS AND DISCUSSION

It can be advanced in the development of the proposal, as was done in Acevedo (2004), and this depends on the teacher's goals in doing the lesson plan. That, for the particular case of the experiment, the intention was the approximation to the concepts involved in algebraic expressions and their development on the basis of a geometric context, which was intended according to the needs of the class.

During the development of the experiment, we recognize that students develop Visual Identification and Perception Conservation skills while doing Visual Processing (PV), as well as performing Visual Discrimination by comparing figures that contain equivalent areas. It was relatively easy to get students to understand the dynamics of the board while using them for similar terms because students visually identified rectangles in the same area. In the same way, students recognized basic operations with algebraic expressions as they used their abilities to recognize spatial positions and spatial relationships to translate the plane of the board and reduce the terms of expressions. The numbers of classes were few for the development of other concepts as a product of polynomials and notable products. However, it is up to the current teacher in the class to continue.

The movement between different types of images allowed the students to identify concepts with representations. That is, the use of visual, kinetic or dynamic representations was coherent, when they recorded in sheets (white or comic), representations of formulas or tokens. It should be noted that the students also used table paper to represent the plans; however, it was required for them to represent the chips in a non-prototypical way, thereby ensuring the constant use of shape properties, as well as the use of instruments for their constructions.

It is also worth mentioning the students' commitment to use materials that help to visualize some abstract concepts of algebra. In addition to the learning they could develop in using their visual skills and processes, students also learned to communicate better with other peers in the group, to make decisions about problems, and to try to negotiate meanings in and out of mathematics.

When proving each of the answers obtained was the order of the day, even though they were students of Pedagogy had to prove different activities, being "presupposed" that they have developed other skills of abstraction throughout college in undergraduate. However, it is necessary to reflect on the training bases of teacher trainers, as well as the pedagogical and disciplinary domain of mathematics, when planning and developing experiences with future teachers, because some of the difficulties in finding solutions to situations may have Originated in visualization skills not developed by their previous formatters and thus, as a consequence, future teachers continue

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reproducing their difficulties through the generations. The reality of future teachers is different. The use of formal vocabulary in the area of mathematics throughout the time of interaction with students requires us to reason differently and to use the properties of that formal mathematics involved in simple construction activities.

CONCLUSSIONS

The analyzes made so far are part of a larger project entitled "Visualization in mathematics Education" which is part of the *Filedumath Research Group*, in which it intends to develop a sequence of activities for the development of visual skills in the training of future mathematics teachers in Universidad Austral de Chile. But it is not easy, for a group of teachers, or for future teachers trained in the exercise paradigm, to see the possibilities that "other" type of activity can be used to reach more transversal learning. Even today, they excel at activities under the exercise paradigm (SKOVSMOSE, 2000, p.66), possibly because it is easier for teachers to pass a series of exercises than to seek a Groups. Besides mathematics and pedagogy in mathematics, there are social learning that is neither written in the curriculum nor quantifiable in the national tests, and they are equally important because they are developed in the contexts of future teachers.

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